Evolutionary computation combined with advanced numerical field analysis for multi-winding transformer design optimization

Marina A. Tsili[1], Eleftherios I. Amoiralis[2], Antonios G. Kladas[1] and Athanassios T. Souflaris[3]
[1]Faculty of Electrical and Computer Engineering, National Technical University of Athens,
9 Iroon Polytechniou Street, 15780 Athens, Greece
[2]Department of Production Engineering and Management, Technical University of Crete, 73100, Greece
[3]Schneider Electric AE, Elvim Plant, 32011, Inofyta, Greece
mtsili@central.ntua.gr

Abstract — In the present article, an efficient finite element method is developed and experimentally validated for the accurate prediction of leakage field and short-circuit impedance in distribution transformers with double primary and secondary windings. The model is used in conjunction with evolutionary and stochastic optimization algorithms in order to derive the optimal winding configuration that meets the technical requirements for the compound multi-winding short-circuit impedance target value.

I. INTRODUCTION

Four winding transformer models have been developed in the literature [1]-[3], mostly devoted to transformers with one primary and three secondary windings. Moreover, significant difficulty in the analytical determination of the equivalent circuit parameters relies in the fact that they are dependent from the main flux that links all the transformer windings, the self leakage inductance of each one of them, as well as the four mutual inductances between them [4]. On the other hand, numerical field analysis techniques as the Finite Element Method (FEM) are indicated for the accurate prediction of the interwinding leakage field and the derivation of these parameters in such composite winding structures.

In the proposed analysis, the results of an efficient FEM model are properly exploited by the application of Design of Experiments (DOE) method and provided as input to various stochastic optimization algorithms. A comparative analysis of the results yields the optimal winding configuration, so as to achieve the desired short-circuit impedance values.

II. DESCRIPTION OF THE MULTI-WINDING TRANSFORMER

The examined 1000 kVA three-phase distribution transformer comprises two independent (primary) High Voltage (HV) and (secondary) Low Voltage (LV) windings per phase, wound around the same core leg, depicted as upper and lower windings in Fig. 1. Upper HV and LV winding of each phase is denoted as HV_1 and LV_1, respectively, while lower HV and LV winding of each phase is denoted as HV_2 and LV_2, respectively. The upper and lower windings of each phase are identical, in order to produce the same voltage level, however, this is not a general requirement for this kind of transformers. HV_1 and HV_2 comprise 1210 turns of copper wire while LV_1 and LV_2 comprise 11 turns of copper sheet. The nominal voltage of HV_1 and HV_2 is 20000V while the nominal voltage of LV_1 and LV_2 is 315V. Four short-circuit tests are simulated, resulting to the following transformer short-circuit impedance values:

U_{HV-LV}: LV_1 and LV_2 are short-circuited, HV_1 and HV_2 operate under nominal current,

U_{HV-LV1}: LV_1 is short-circuited, LV_2 winding is open-circuited, HV_1 and HV_2 operate under nominal current,

U_{HV-LV2}: LV_1 is open-circuited, LV_2 winding is short-circuited, HV_1 and HV_2 operate under nominal current,

$U_{LV1-LV2}$: LV_1 winding is short-circuited, LV_2 winding operates under nominal current, HV_1 and HV_2 are open-circuited.

In the case of the considered transformer, where HV_1 and HV_2 as well as LV_1 and LV_2 are identical, $U_{HV-LV1}= U_{HV-LV2}$.

III. TRANSFORMER FEM MODEL

A. Model configuration

For the accurate calculation of the interwinding leakage field, an efficient transformer FEM model has been developed, based on a particular scalar potential formulation, enabling the 3D magnetostatic field analysis [5]. Special consideration is given to the detailed winding geometry, taking into account the elliptic shape of the winding corners and the cooling ducts dimensions [6]. The model comprises the upper and lower LV and HV windings of one phase, as well as the small and large iron core that surrounds them.

B. Validation by measurements

Table I lists the predicted and measured values for the four short-circuit tests described in the previous Section, as well as their deviation, indicating the accuracy of the FEM model. It must also be noted that all results are obtained by a relatively coarse mesh, with total execution time less than 1 min in a PC of medium computational capability.

TABLE I
COMPARISON OF COMPUTED AND MEASURED SHORT-CIRCUIT IMPEDANCE VALUES FOR THE 1000KVA TRANSFORMER

Short-circuit Impedance	*Computed by FEM (%)*	*Measured (%)*	*Deviation (%)*
U_{HV-LV}	7.54	7.20	4.74
$U_{HV-LV1} = U_{HV-LV2}$	6.35	6.00	5.82
$U_{LV1-LV2}$	9.88	10.00	1.20

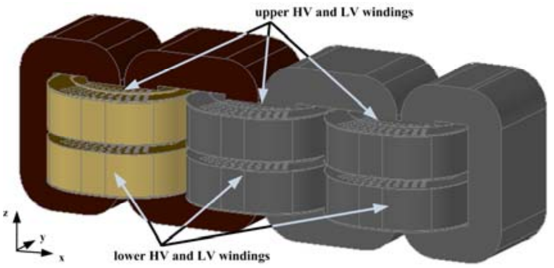

Fig. 1. Active part configuration of the examined multi-winding distribution transformer (the colored components correspond to the one-phase part modeled in FEM).

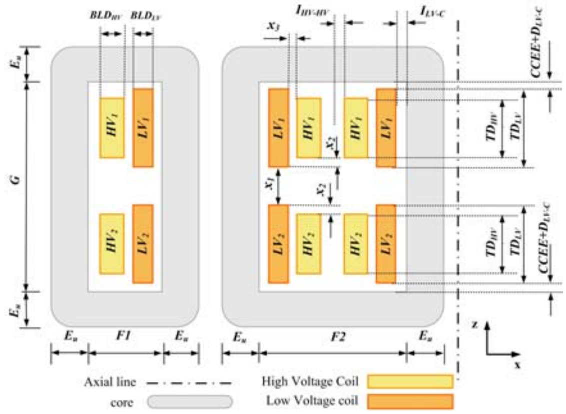

Fig. 2. 2D representation of one-half of the transformer active-part (xz-plane), illustrating the four winding geometry optimization variables.

IV. GEOMETRY OPTIMIZATION OF WINDINGS

A. Mathematical Formulation

The interwinding leakage field is mainly influenced by the distance between the lower and upper windings (vertical gap) and the inner and outer windings (horizontal gap). These gaps, also depicted in Fig. 2 are the design variables of the geometry optimization problem. The goal is to achieve the optimum balance between these values, in order to minimize the difference between the specified and designed short-circuit impedances. The analytical expression of the objective function is given by:

$$F = \sqrt{(U_1 - U_1^{spec})^2 + (U_2 - U_2^{spec})^2 + (U_3 - U_3^{spec})^2} \,. \quad (1)$$

where $U_1 = U_{HV\text{-}LV}$, $U_2 = U_{HV\text{-}LV1} = U_{HV\text{-}LV2}$ and $U_3 = U_{LV1\text{-}LV2}$, while U_1^{spec}, U_2^{spec} and U_3^{spec} are the respective specified values. The upper bounds x_1^{max}, x_2^{max} and x_3^{max} of x_1, x_2 and x_3, imposed by the geometrical restrictions of the active part (Fig. 2), are defined by the following equations:

$$x_1^{max} = G - (2 \cdot TD_{LV} + 2 \cdot CCEE + 2 \cdot D_{LV-C}) \,. \quad (2)$$

$$x_2^{max} = 0.5 \cdot (TD_{LV} - TD_{HV}) \,. \quad (3)$$

$$x_3^{max} = 0.5 \cdot F2 - (BLD_{HV} + BLD_{LV} + I_{HV-HV} + I_{LV-C}) \,. \quad (4)$$

B. Design of Experiments

Design of Experiments is used for the process of planning, designing and conducting a series of calculations based on the FEM model so that valid conclusions can be drawn effectively for the objective short-circuit impedance values. Full factorial experiments are conducted and proper regression models are used to approach the relationship between the response value (namely, the short-circuit impedance) and the design variables [7]. The response function is used as input to various optimization algorithms for the calculation of the optimal winding geometry.

C. Optimization Methods

Various stochastic algorithms are used and their results are compared in Table II. Due to the stochastic nature of the algorithms, converging to different optimum designs for different executions, the mean optimum objective function value of ten runs of each algorithm is included in Table II. Moreover, the percentage deviation of each short-circuit component from the specified value (*dU_1*, *dU_2* and *dU_3*), expressed as percentage of the specified value, as well as the average value of these three deviations, is included in Table II. According to these results, Genetic Algorithms produce the best solution in terms of deviation of each component of (1) from the specified value. It must be noted that, although Simulated Annealing results to an acceptable value of (1), the deviation of U_2 and U_3 from the specified value exceeds 10%, a value which is often imposed as an upper limit of the tolerance in the deviation of short-circuit impedance according to international standards [8].

More optimization results will be presented in the full paper, and other transformer ratings will be included in the analysis, providing significant conclusions for the optimization process.

TABLE II

OPTIMIZATION RESULTS FOR U_1^{spec} =7.2%, U_2^{spec} =6%, U_3^{spec} =10%

Method	*F(%)*	*dU_1 (%)*	*dU_2 (%)*	*dU_3 (%)*	*Mean deviation (%)*
Monte-Carlo	0.77	2.64	6.33	6.40	5.12
Simulated Annealing	1.92	9.17	13.33	16.10	12.87
Genetic Algorithm	0.53	2.64	2.17	4.80	3.20
Pattern Search	0.75	1.39	0.83	7.40	3.21

V. REFERENCES

[1] J. Wang, A.F. Witulski, J.L. Vollin, T.K. Phelps, G.I. Cardwell, “Derivation, calculation and measurement of parameters for a multi-winding transformer electrical model,” in *Proc. 14th Annual Applied Power Electronics Conference and Exposition (APEC '99)*, vol. 1, pp. 220-226, 1999.

[2] C. P. Schultz, “A contribution to the theory of multiple winding transformers,” in *Proc. Southeastcon 2009*, pp. 175-177, 2009.

[3] V. A. Niemela, “Leakage-impedance model for multiple-winding transformers,” in *Proc. IEEE 31st Annual Power Electronics Specialists Conference, PESC 2000*, vol.1, pp.264-269.

[4] M. Heathcote, *The J&P Transformer Book*, 12th ed., Oxford: Newnes, 1998.

[5] A. Kladas, J. Tegopoulos, "A new scalar potential formulation for 3D magnetostatics necessitating no source field calculation," *IEEE Trans. Magn.*, vol. 28, pp. 1103-1106, 1992.

[6] M.A. Tsili, A.G. Kladas, P.S. Georgilakis, A.T. Souflaris, D.G. Paparigas, “Advanced design methodology for single and dual voltage wound core power transformers based on a particular finite element model,” *Electric Power Systems Research*, Vol. 76, pp. 729-741, Apr. 2006.

[7] J. Antony, *Design of Experiments for Engineers and Scientists*, Elsevier Science and Technology Books, Oct. 2003.

[8] IEC 60076-1, Power Transformers—Part 1: General 2000.